Normal Distributions

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Three types of Continuous R.V.

Conti. R.V.

- General and Sampling Dist. — Normal and Standard Normal
- General Dist. — Rectangular, Exponential, Gamma, Erlang, Weibull Rayleigh, Lognormal, Beta
- Sampling Dist. — Chi-Square (χ^2), T, F

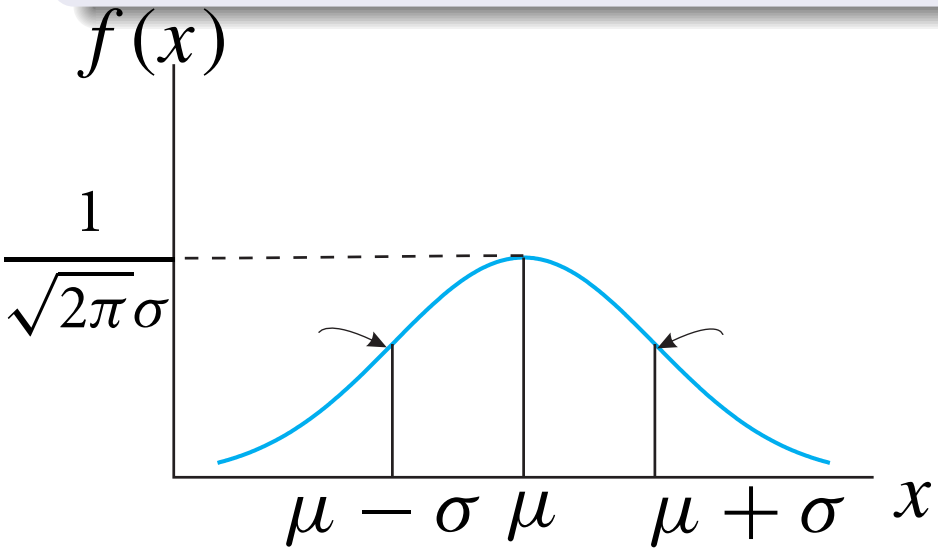
- Distinguish 3 types of continuous distributions.

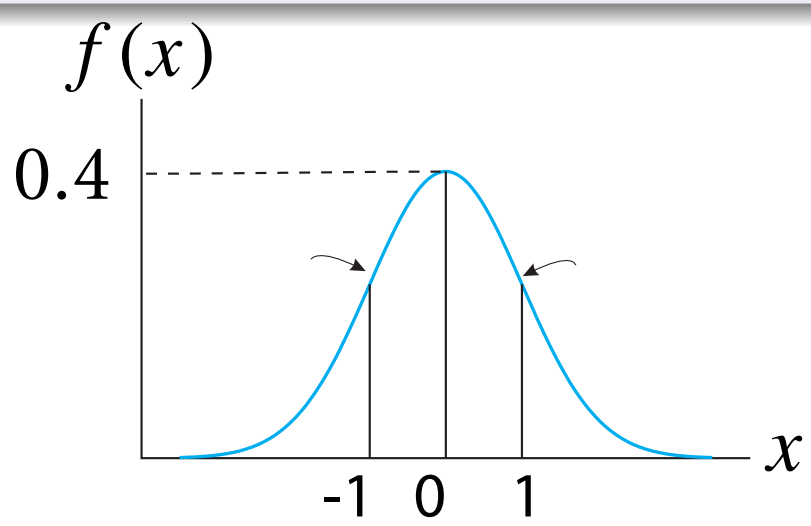
Normal Dist.

$X \sim \text{normal}(\mu, \sigma^2)$, or $X \sim \text{N}(\mu, \sigma^2)$

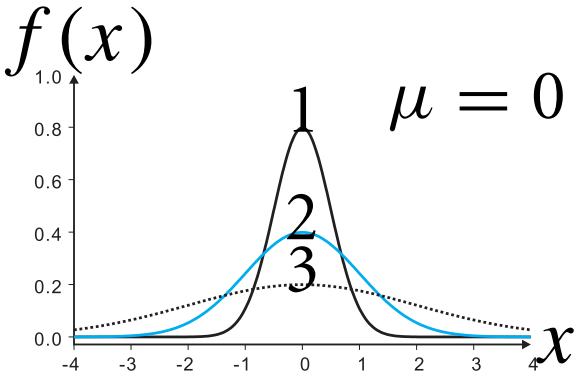
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \text{e}^{(-1/2)[(x-\mu)/\sigma]^2}, \quad -\infty < x < \infty,$$

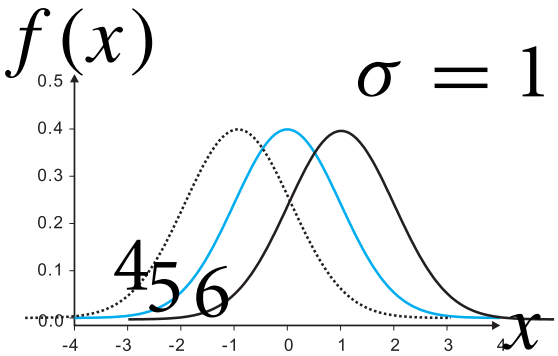
$$\pi = 3.1415\ldots, \text{e} = 2.7182\ldots, -\infty < \mu < \infty, \sigma \geq 0.$$





Moments of Normal Dist.





	μ	σ	Mean	Median	Mode	Variance	Skewness	Kurtosis
1	0	0.5	0	0	0	0.25	0	3
2	0	1	0	0	0	1	0	3
3	0	2	0	0	0	4	0	3
4	-1	1	-1	-1	-1	1	0	3
5	0	1	0	0	0	1	0	3
6	1	1	1	1	1	1	0	3

- The first 4 moments are ...

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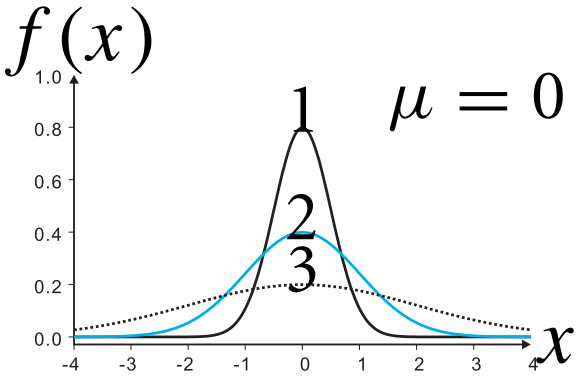

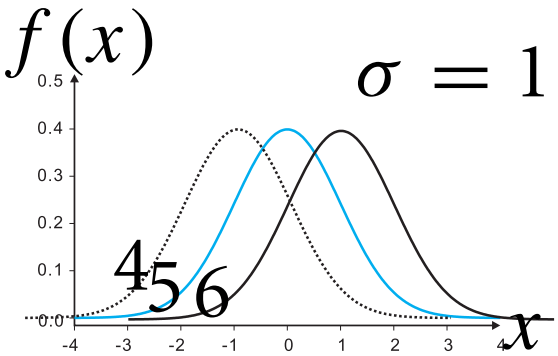

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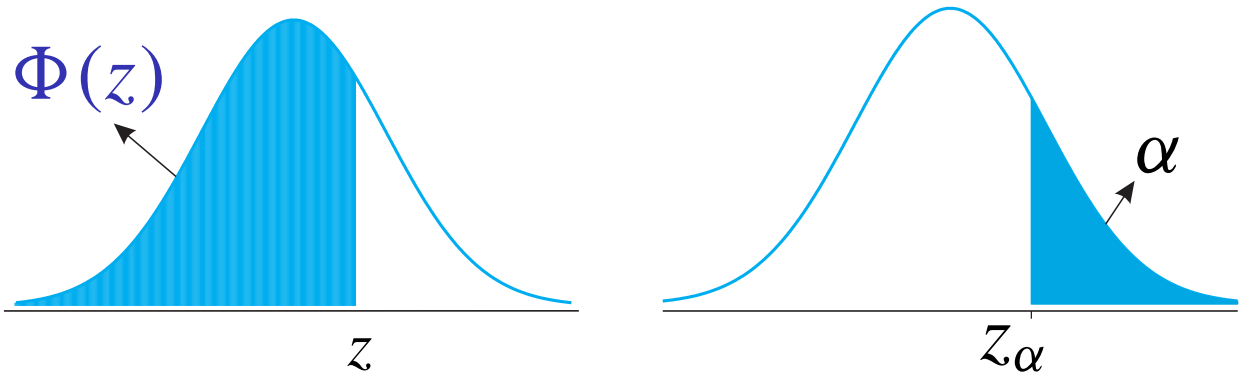
Area Under N(0,1)



(a) z 與 $\Phi(z)$ (b) α 與 z_α

- $\mathrm{P}(-1 \le Z \le 1) = \Phi(1) - \Phi(-1) \simeq 0.6827$
- $\mathrm{P}(-2 \le Z \le 2) = \Phi(2) - \Phi(-2) \simeq 0.9545$
- $\mathrm{P}(-3 \le Z \le 3) = \Phi(3) - \Phi(-3) \simeq 0.9973$
- Illustrate $\Phi(z)$ and z_α

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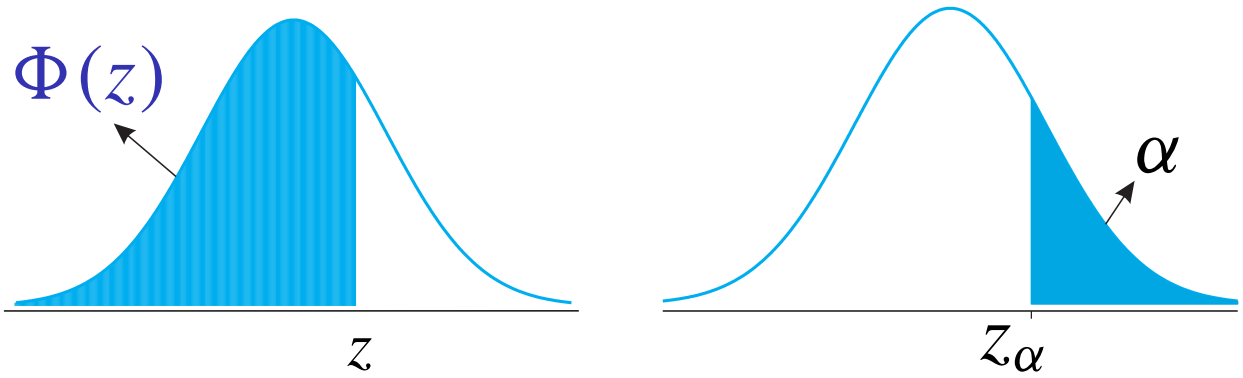

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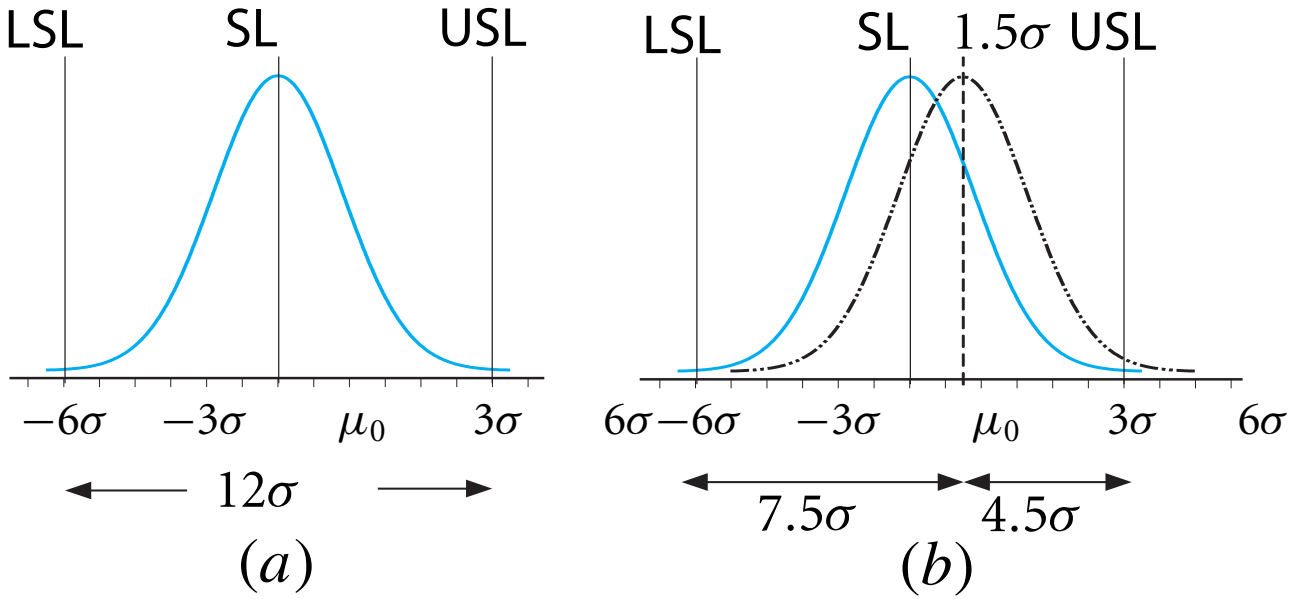
常態分配應用 (CLT))

請看 CLT 講義

常態分配應用 (CLT))

麵包司傅作弊 (open ppt)

常態分配應用 (6σ)



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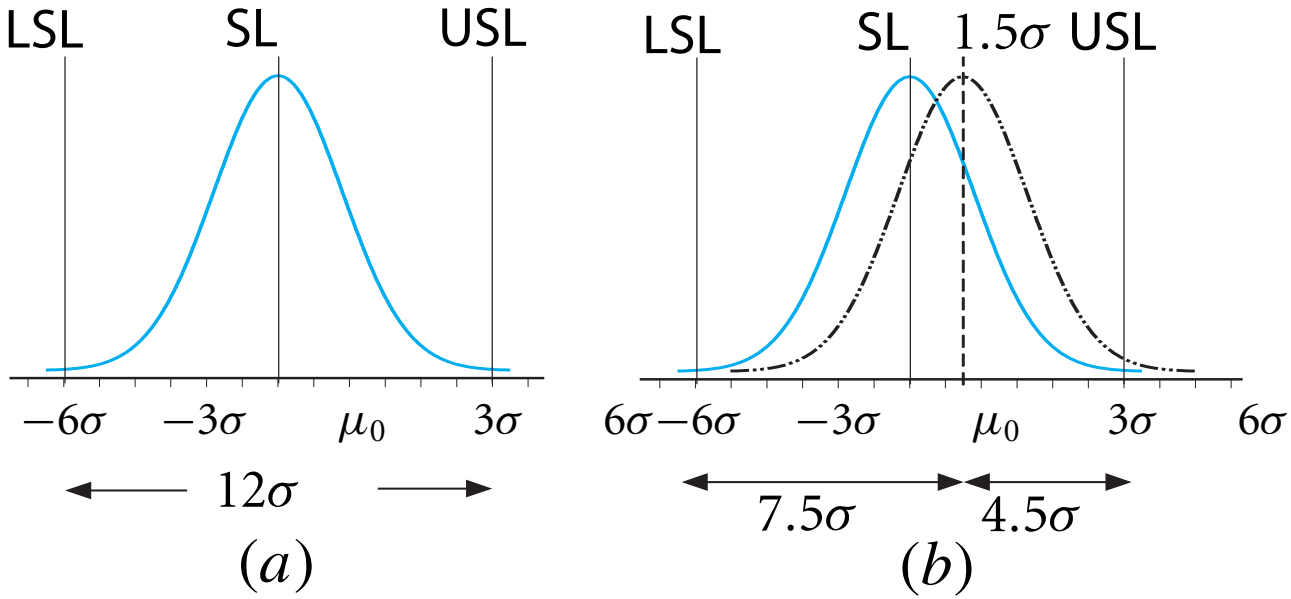

- 請你解釋 Motorola 提出的 6σ
- P($X >$ USL 或 $X <$ LSL$|\mu = \mu_0) = 0.002 \times 10^{-6}$;「0.002 ppm」;
  「一百萬個產品中有 0.002 個不良品」
- P($X >$ USL 或 $X <$ LSL$|\mu = \mu_0)3.4 \times 10^{-6}$;「3.4 ppm;
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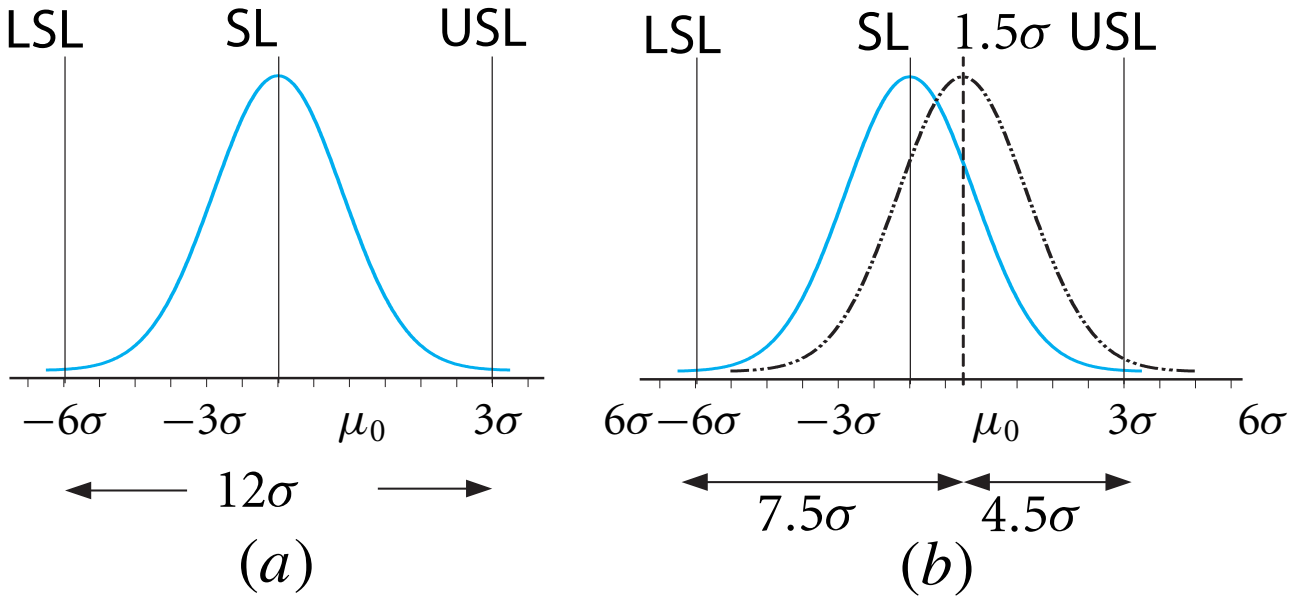

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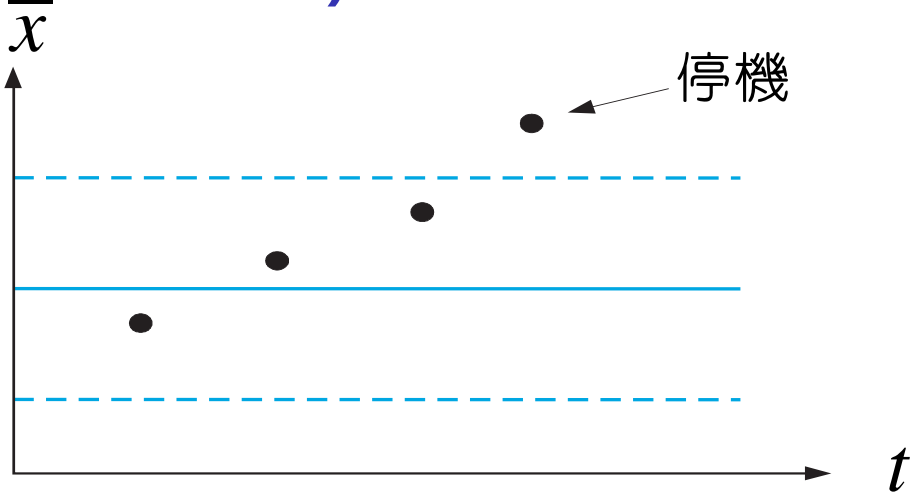
常態分配應用 ($\overline{X}$ 管制圖)

- $\overline{X}$ Chart
  - UCL
  - CCL
  - LCL

$$\text{UCL} = \mu_0 + k\sigma_{\overline{X}}$$

$$\text{CCL} = \mu_0$$

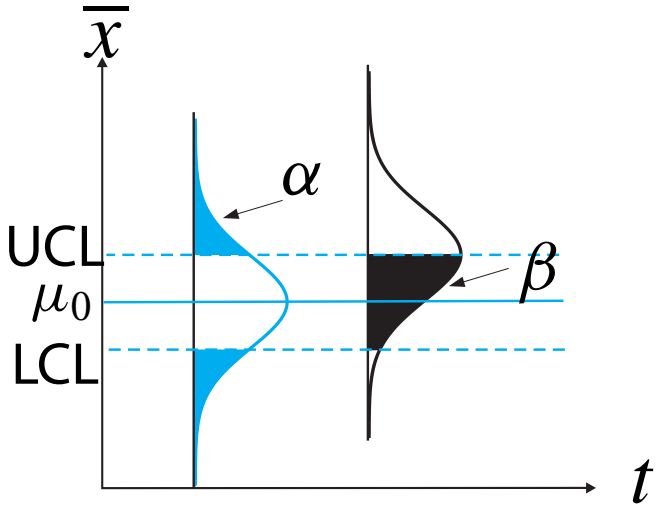
$$\text{LCL} = \mu_0 - k\sigma_{\overline{X}}$$



- Two Types of Errors

$H_0: \mu = \mu_0$ (製程正常)

$H_1: \mu \neq \mu_0$ (製程偏移)



- Distinguish (Type I error, α) and (Type II error, β)